It is evident from the figure that with a sufficiently long time of action of the pulse τ_0, $\eta_*^0 \to 1$. With short times, the value of η_*^0 is considerably greater than unity, i.e., the estimate obtained is not trivial, coincident with the critical force under static loading N_e. Rather, it is significantly greater than this force, which makes it possible to obtain a substantially higher permissible compressive force during shock loading than under static loading. Consequently, the structure can withstand larger loads than originally believed. A determination should be made of the boundaries of the parameters which, when approached, signify that the results obtained here have become unreliable.

At $\tau_* < 2$-3, the results may prove unreliable due to failure to account for the finite rate of propagation of the compressive force in the rod.

When $W_0/\varepsilon_0 < 10$, the results become unreliable due to representation (2.8) and the fact that the remaining part of the sum was ignored. On the other hand, with large values of W_0 the results become unreliable because we examined a linear equation of rod bending.

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METHODS OF SOLVING CONTACT THERMOELASTICITY PROBLEMS WITH ALLOWANCE FOR THE WEAR OF INTERACTING SURFACES

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1. Assume that a heavy cylindrical stamp is pressed into a rough, elastic (G, ν) layer with a large thickness, h. A force P, which is constant in time, is applied with the eccentricity e for each unit length of the stamp. The stamp moves at a constant velocity V along its generatrix; it is assumed that its area of contact with the layer has the width $2a(ha^{-1} \gg 1)$ and does not change in the course of time (see Fig. 1). This involves wear of the layer surface, which is accompanied by heat release in the region of contact. We assume that the stamp itself is not subject to wear. Coulomb friction forces arise in the region of contact [1, 2],

$$\tau_{yz} = (k_1 + k_2T)q, \tag{1.1}$$

where k_1 and k_2 are constants, T is the temperature in the region of contact, and $q = q(x, t)$ is the contact pressure.

The condition of contact for solids 1 and 2 is written as follows:

$$v_1 + v_2 + v_3 = -[\delta(t) + \alpha(t)x - f(x)] \quad (|x| \leqslant a), \tag{1.2}$$

where v_1 is the displacement of the elastic layer's upper boundary due to the crushing of roughnesses, v_2 is the elastic deformation of the layer's surface, v_3 is the displacement of the y = 0 boundary of the layer due to its wear, $\delta(t) + \alpha(t)x$ is the rigid displacement of

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the stamp under the action of force P, and f(x) is the shape of the stamp's base.

We assume that the displacement v_1 depends linearly [3] on the contact pressure q:

$$v_1 = -lq \tag{1.3}$$

where l is a constant which characterizes the degree of roughness of the solids in contact. The displacement v_2 is defined by [4]

$$v_2 = -\frac{1}{\pi\theta}\int_{-a}^{a} q(\xi, t)(-\ln|\xi - x| + d)\, d\xi,\ \theta = \frac{G}{1-\nu}, \tag{1.4}$$

where $d = \ln h + \alpha_0$; $\alpha_0 = -0.527$ for $\nu = 0.3$.

Before we proceed to determine the displacement v_3, we should note the following fact. We denote by A the work of the friction force τ_{yz} along the sliding path of the stamp. It can obviously be represented in the following form:

$$A = A_1 + A_2,\ A_1 = n_1 A,\ A_2 = n_2 A,\ n_1 + n_2 = 1, \tag{1.5}$$

where A_1 is the work of the friction force expended on wear of the layer surface, and A_2 is the work of the friction force expended on heat release in the contact region.

Considering the latter relationships and expression (1.1), we write the displacement v_3 in the following form [5]:

$$v_3 = -n_1 V \int_0^t m[T(x,\tau)][k_1 + k_2 T(x,\tau)]\, q(x,\tau)\, d\tau, \tag{1.6}$$

where m is the coefficient of wear intensity, which is a function of the temperature.

Thus, the integral equation for determining the contact pressure in correspondence with (1.2)-(1.4) and (1.6) is given by

$$lq(x,t) + \frac{1}{\pi\theta}\int_{-a}^{a} q(\xi,t)(-\ln|\xi - x| + d)\, d\xi + n_1 V\int_0^t m[T(x,\tau)][k_1 + k_2 T(x,\tau)]\, q(x,\tau)\, d\tau = \delta(t) + \alpha(t)x - f(x) \quad (|x| \leqslant a,\ 0 \leqslant t \leqslant \Theta < \infty). \tag{1.7}$$

We assume here that the value of Θ is sufficiently large, but such that $\delta(\Theta) + \alpha(\Theta)a$ has the order of magnitude of displacement in linear elasticity theory.

In solving thermal conductivity problems for solids 1 and 2, we shall assume that they are unbounded in the direction perpendicular to the contact plane. We neglect the term $\partial T/\partial t$, in the equation of thermal conductivity, since the wear process occurs relatively slowly in time. In this case, the thermal conductivity equations must be added to the integral equation (1.7):

$$\Delta T_i(x, y, t) = 0 (i = 1, 2,\ \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2) \tag{1.8}$$

for the boundary conditions

$$\alpha_1 \partial T_1/\partial y - \alpha_2 \partial T_2/\partial y = n_2 \beta V(k_1 + k_2 T)q \quad (y = 0,\ |x| \leqslant a); \tag{1.9}$$

$$T_1 = T_2 \quad (y = 0,\ |x| \leqslant a); \tag{1.10}$$

$$(-1)^i \partial T_i/\partial y + \varkappa_i T_i = 0 \ (y = 0,\ |x| > a,\ i = 1, 2); \tag{1.11}$$

the gradients of T_i vanish at infinity. Here $T_i(x, y, t)$ is the temperature in the i-th solid, α_i are the thermal conductivity coefficients, $\varkappa_i$ are the heat exchange coefficients, and β is the thermal equivalent of mechanical work.

For closure in the statement of the problem, the conditions of quasiequilibrium of the stamp on the layer must be added to Eqs. (1.7)-(1.11):

$$P = \int_{-a}^{a} q(x,t)\, dx, \quad Pe = \int_{-a}^{a} xq(x,t)\, dx. \tag{1.12}$$

2. It should be noted that the above statement of the problem necessitates the solution of the nonlinear system (1.7)-(1.12). However, the problem can be simplified in the following manner.

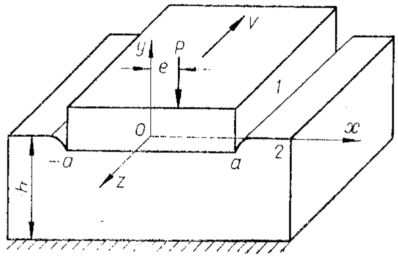

Fig. 1

We introduce the coefficient of thermal flux separation in the contact region by assuming that

$$\partial T_1/\partial y = -\alpha\partial T_2/\partial y \quad (y = 0,\ |x| \leqslant a). \tag{2.1}$$

The thermal conductivity problem (1.8)-(1.11) is then separated into two independent problems with the following boundary conditions for y = 0:

$$\alpha_1[1 + \alpha_2(\alpha_1\alpha)^{-1}]\partial T_1/\partial y = n_2\beta V(k_1 + k_2T_1)q$$
$$(|x| \leqslant a),\ -\partial T_1/\partial y + \varkappa_1T_1 = 0 \quad (|x| > a); \tag{2.2}$$

$$-\alpha_1\alpha[1 + \alpha_2(\alpha_1\alpha)^{-1}]\partial T_2/\partial y = n_2\beta V(k_1 + k_2T_2)q$$
$$(|x| \leqslant a),\ \partial T_2/\partial y + \varkappa_2T_2 = 0\ (|x| > a), \tag{2.3}$$

where T_i (i = 1, 2) vanish at infinity.

After problems (1.8), (2.2), and (2.3) have been solved, the coefficient of thermal flux separation α must be found from the boundary condition (1.10), written in integral form.

$$\bar{T}_1 = \bar{T}_2,\quad \bar{T}_i(t) = \frac{1}{2a}\int_{-a}^{a} T_i(x, 0, t)\,dx. \tag{2.4}$$

For the sake of simplicity, we furthermore assume that, instead of q(x, t), its mean value $\bar{q}(t) \equiv \bar{q} = P(2a)^{-1}$ appears in Eqs. (2.2) and (2.3). We then note that the functions T_i, defined in accordance with (1.8), (2.2), and (2.3), and, thus, also the $\bar{T}_i$ functions (2.4), no longer depend on the time.

Using the integral Fourier transform with respect to the longitudinal coordinate x, we reduce problems (1.8), (2.2), and (2.3) to the solution of the integral equations

$$p_i(x) + \frac{g_2^{(i)}}{\pi}\int_{-a}^{a} p_i(\xi)\,d\xi \int_0^\infty \frac{\cos u\varkappa_i(\xi - x)}{u+1}\,du = g_1^{(i)} \tag{2.5}$$
$$(|x| \leqslant a,\ i = 1,\ 2).$$

Here, $p_i(x)$ are related to $T_i(x, y)$ for y = 0 by the expression

$$T_i(x, 0) = -\frac{1}{\pi}\int_{-a}^{a} p_i(\xi)\,d\xi \int_0^\infty \frac{\cos u\varkappa_i(\xi - x)}{u+1}\,du, \tag{2.6}$$

while the constants $g_1^{(i)}$ and $g_2^{(i)}$ have the following form:

$$g_1^{(i)} = \frac{n_2k_1\beta V\bar{q}\alpha^{1-i}}{\alpha_1\left[1 + \alpha_2(\alpha_1\alpha)^{-1}\right]},\quad g_2^{(i)} = \frac{n_2k_2\beta V\bar{q}\alpha^{1-i}}{\alpha_1\left[1 + \alpha_2(\alpha_1\alpha)^{-1}\right]} - \varkappa_i.$$

The integral equation (2.5) is uniquely solvable in the space of continuous functions $C(-a, a)$ for $k_2 > 0$, which can be ascertained by using the method given in [6], while its solution can be found, for instance, by expanding the function $p_i(x)$ in a series with respect to Legendre polynomials. If $k_2 < 0$, there obviously exists a denumerable set of parameter values$[g_2^{(ij)}]$ (i = 1, 2, j = 1, 2, ...), such that the homogeneous integral equation (2.5) is solvable in $C(-a, a)$, i.e., in this case, there is a denumerable set of stamp velocities which entail loss of quasistationary thermoelastic stability of the system. This denumerable set of constants $[g_2^{(ij)}]$ can be determined by using the Ritz method [8].

If $g_2^{(i)}$ is not a point in the spectrum of the integral operator on the left-hand side of (2.5), we simplify Eq. (1.7) by substituting in it the mean temperature value $\bar{T}$ found from relationships (2.5) and (2.6) instead of T(x, t). We then have

$$lq(x,t)+\frac{1}{\pi\theta}\int_{-a}^{a} q(\xi,t)(-\ln|\xi-x|+d)\,d\xi+n_1 m(\overline{T})V(k_1+k_2\overline{T})\int_0^t q(x,\tau)\,d\tau=$$
$$=\delta(t)+\alpha(t)x-f(x)\quad(|x|\leqslant a,\ 0\leqslant t\leqslant\Theta<\infty).$$

The solution of the latter integral equation for conditions (1.12) can be obtained by using the method described in [8, 9]. Thus, for a sufficiently long time of wear, we obtain

$$q(x,t)=\overline{q}(1+3ex/a^2),$$
$$\delta^{\cdot}(t)=n_1 m(\overline{T})V(k_1+k_2\overline{T})\overline{q},\ \alpha^{\cdot}(t)=3n_1 m(\overline{T})V(k_1+k_2\overline{T})e\overline{q}/a^2.$$

In conclusion, it should be noted that the coefficients n_1 and n_2 must be determined experimentally for each specific combination of contiguous solids.

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NUMERICAL ANALYSIS OF FRACTURE IN PLATES UNDER THE ACTION OF IMPACT LOADS

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Calculation of fracture in solids with limited dimensions under the action of impact loads can be considered by formulating a macrofailure criterion. Fulfillment of such a criterion in a particle of the material signifies its breakdown. In the presence of a complex wave interference pattern in the numerical solution, such a criterion is satisfied in entire regions. This requires formulation of a model of the fractured solid in numerical calculations [1, 2].

There is another approach to calculating the disintegration of solids under detonation or impact loads, which is based on the porous solid model [3-6]. We shall write below the basic equations of a compressible elastoplastic medium with pores and investigate numerically the disintegration process in plates under the action of dynamic loads.

1. We shall assume that spherical defects with the radius α_0 exist in the solid. We introduce a spherical coordinate system with the origin in the spherical cavity, whose present radius is denoted by α. Assume that the stress $\sigma_r = -p$ acts at the distance b from the cavity. The porosity is characterized by

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